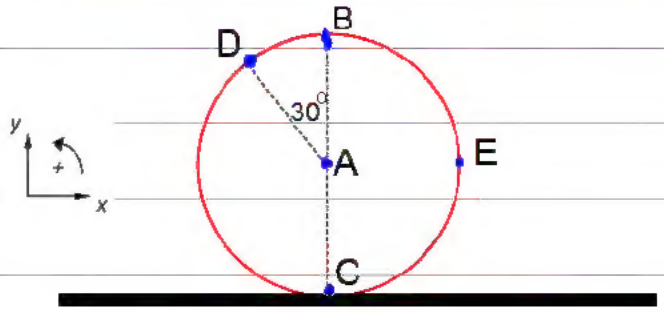
Meriam 5/5: Instantaneous Center of Zero Velocity



Given: $\vec{V}_A = 25\,\hat{i}$ m/s

$R_{wheel} = 0.275$ m

no slipping

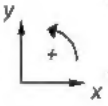
Find $\vec{V}_D$

Answer:

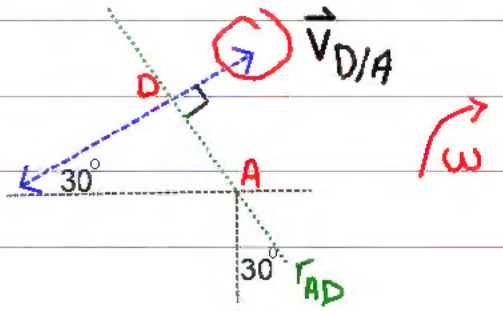
$\vec{V}_A = 25\,\hat{i}$

$\vec{V}_C = 0$ (no skidding)

$|\omega| = |\vec{V}_C - \vec{V}_A| / r_{AC} = 25/0.275 = 90.91$



$\rightarrow \omega = -90.91\ s^{-1}$ $(= -90.91\,\hat{k}\ s^{-1})$



$\vec{V}_D = \vec{V}_A + \vec{V}_{D/A} = 25\,\hat{i} + |\omega\, r_{AD}|$

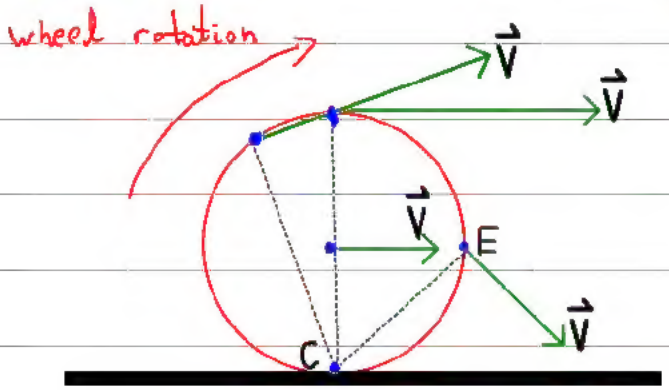
$= 25\,\hat{i} + (90.91)(0.275)[\cos 30\,\hat{i} + \sin 30\,\hat{j}]$

$\vec{V}_D = 46.6\,\hat{i} + 12.6\,\hat{j}$ m/s

similarly, $\vec{V}_B = \vec{V}_C + \vec{V}_{B/C} = |\omega\, r_{BC}| = (90.91)(0.55)\hat{i} = 50\,\hat{i}$ m/s

or $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = \vec{V}_A + |\omega\, r_{AB}| = 25\,\hat{i} + (90.91)(0.275)\hat{i} = 50\,\hat{i}$ m/s

$\vec{V}_E = \vec{V}_A + \vec{V}_{E/A} = \vec{V}_A + |\omega\, r_{AE}| = 25\hat{i} + (90.91)(0.275)(-\hat{j}) = 25\hat{i} - 25\hat{j}$ m/s



Note: $\vec{V}_E = \vec{V}_C + \vec{V}_{E/C}$

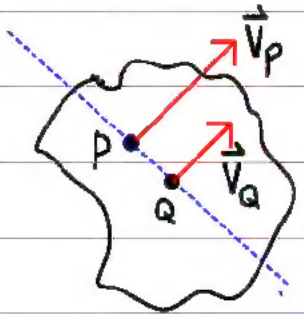
$\rightarrow$ Point C is an instantaneous center of zero velocity (i.c.)

Does every 2-D body have an i.c.?

- **Possibility (a)**: Translation with no rotation, $\omega = 0$
  → all points have the same $\vec{V}$ → no i.c.

- **Possibility (b)**: $\omega \neq 0$

Consider an arbitrary point P with velocity $\vec{V}_P$

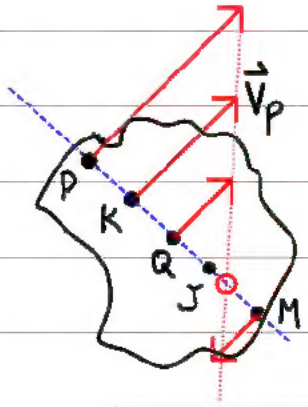


Draw a line through point P, perpendicular to $\vec{V}_P$

Consider a point Q anywhere on this line

$$\vec{V}_Q = \vec{V}_P + \vec{V}_{Q/P}$$

perpendicular to PQ, magnitude $|\omega r_{PQ}|$
→ parallel to $\vec{V}_P$



Select several more points on blue line

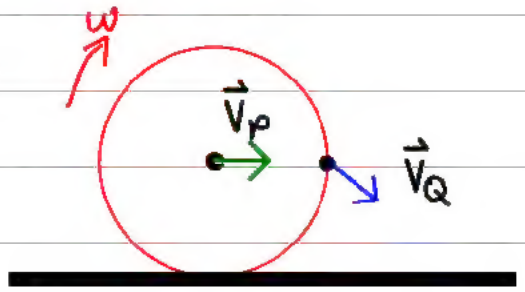
Interpolate/extrapolate to find i.c. ○

(i.c may be outside of boundary, e.g. spinning donut.)

→ Every 2D body with non-zero ω has an i.c.

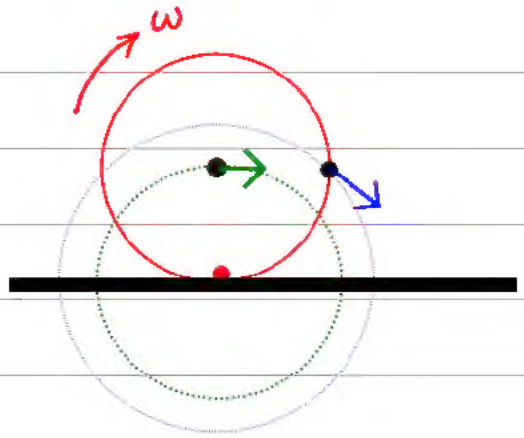
How can we quickly find the i.c of any 2D body?

- Suppose we know direction of velocity of any 2 points P & Q



- P and Q are both going in a circle around the i.c at this instant.

→ The i.c. must lie on a line going through P, perpendicular to the velocity of P.
Ily for point Q.

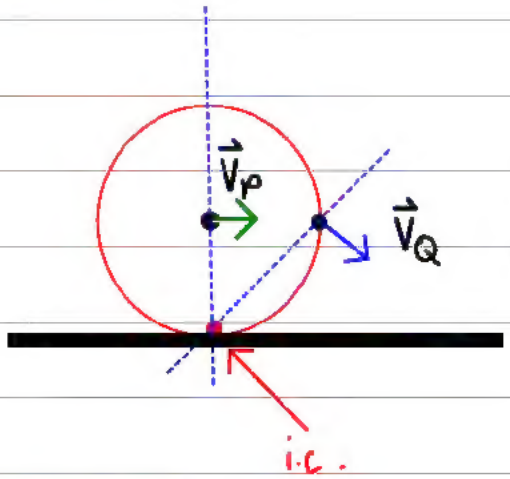


→ Intersection of those lines is the instantenous center of zero velocity (i.c.)

To find the i.c:

1/ Find 2 points whose velocity direction is known
2/ Draw lines through those points, perpendicular to $\vec{V}$
3/ Intersection of the two lines is

Instantaneous Center (i.c.) of zero velocity



Eg.

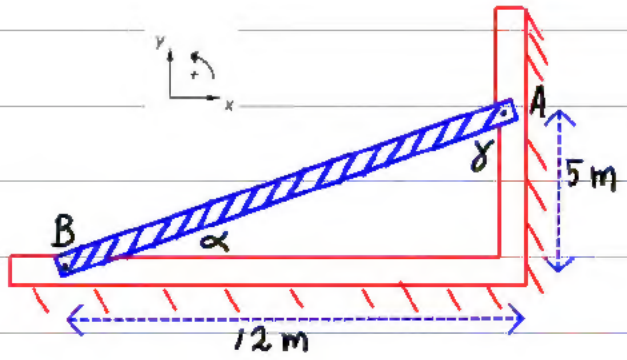
Given:

A can move vertically

B can move horizontally

$\vec{V}_A = -8\hat{j}$ m/s

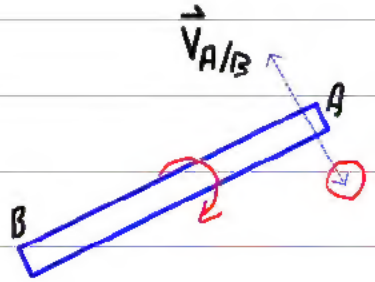
Find ω



Method 1:

$\alpha = \tan^{-1}(5/12) = 22.6°$

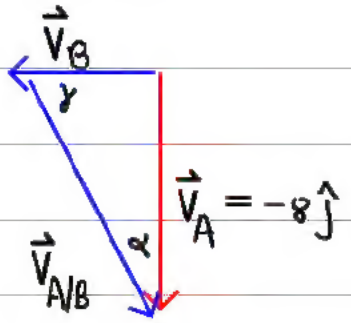
$\gamma = 90° - 22.6° = 67.4°$



$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$

where $\vec{V}_B$ is horizontal ⟷

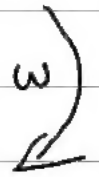
$\vec{V}_{A/B}$ is ⊥ to r_{AB} ↕



→ $|\vec{V}_{A/B}| = |\vec{V}_A| / \sin\gamma = 8 / \sin 67.4° = 8.66$ m/s

$|\omega| = |\vec{V}_{A/B}| / r_{AB} = \frac{8.66}{13} = 0.666\ s^{-1}$

$\omega = -0.666\ s^{-1}$



(get +/− direction from diagram)

or/

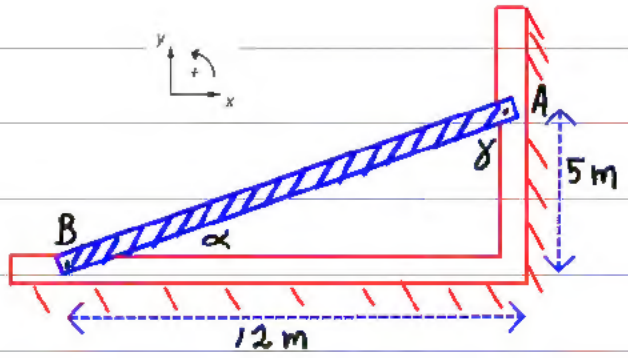
Given:

A can move vertically

B can move horizontally

$\vec{v}_A = -8\hat{j}$ m/s

Find ω



i.c is at point C

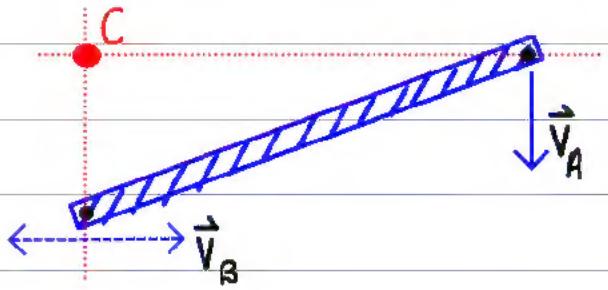
$|\vec{v}_A| = |\omega r_{AC}|$

$8 = |(\omega)(12)|$

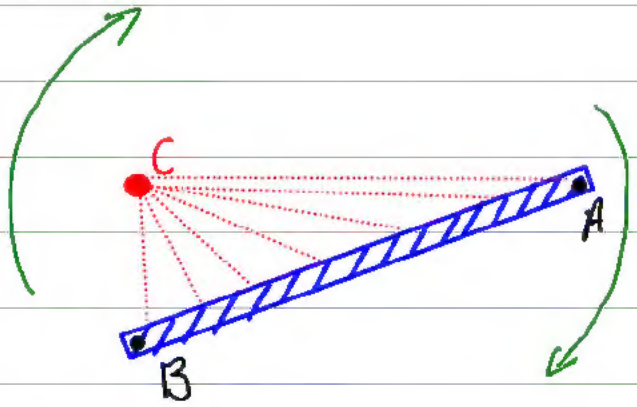
$|\omega| = 0.667\ s^{-1}$

$\omega = -0.667\ s^{-1}$

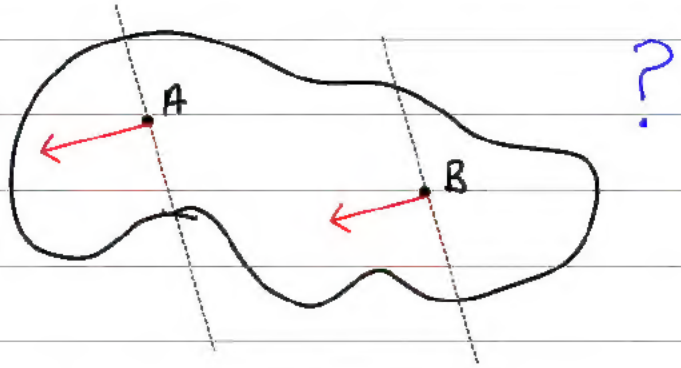
(get +/− direction from diagram)



At this instant, bar AB is behaving as if it were joined to C by several stiff wires, and rotating about C



Eg. Suppose we have a situation where we can't find the i.c.



$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$\rightarrow \vec{v}_A = \vec{v}_B$; $|\vec{v}_{A/B}| = |\omega r_{AB}| = 0$

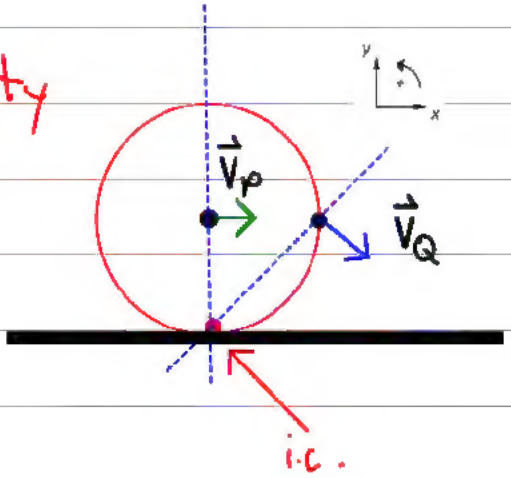
Summary: General Plane Motion

- $\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$
  $= \vec{V}_B + |\omega r_{AB}|$

$\perp$ to r_{AB}

where $$|\omega| = \frac{|\vec{V}_P - \vec{V}_Q|}{r_{PQ}}$$

- Instantaneous Center (i.c.) of zero velocity



Next Lecture:

- Relative acceleration: Meriam 5/6
- Sample Problems
- Read / Review Sections 5/1 – 5/6